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Research article

MATHEMATICAL MODEL TO MONITOR THE DEPOSITION OF VOID RATIO AND DISPERSION OF PHOSPHORUS INFLUENCE IN SALMONELLA GROWTH RATE IN COARSE AND GRAVEL FORMATION IN BORIKIRI, RIVERS STATE OF NIGERIA

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Abstract

Mathematical model to monitor the deposition of void ratio and dispersion on phosphorus deposition influence in salmonella growth rate in coarse and gravel formation has been expressed. The model were develop to express the influence of void ratio in soil on the deposition of phosphorous influencing the migration of salmonella in coarse and gravel formation, the degree of void ratio express the rate of tortuosity deposition in the formations, the study area is in deltaic environment, these condition generate high degree of flow paths from tortuosity through high degree of void ratio in the formation, the formation are influence by this expressed parameters in the system, thus develop fast migrations of salmonella in the formations, fast migration of solute in gravel and coarse formation in the study area are confirmed to deposit high hydraulic conductivity, this implies that deposition of phosphorous will be high in gravel and coarse formation, high degree of void ratio will definitely increase the concentration of salmonella, such situation has a reflection on the increase of the microbes through the deposition of phosphorous. To monitor the pollution transport from salmonella and deposition of phosphorous, mathematical model were the best approach to monitor the rate of these pollution transport in gravel and coarse sand formations, the model were derived through formulated mathematical governing equation, parameters that express the deposition of phosphorous and salmonella concentration in gravel and coarse formations were considered in the system, the rate of void ratio and salmonella migration are on the exponential phase, the model were expressed mathematically on such condition confirmed in the study area, the developed model will be useful to experts to monitor the stated parameters under study in the assessments of soil and water pollution in the study area.

Keywords: mathematical model, void ratio, Deposition of phosphorous, salmonella and gravel formations.

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1. Introduction

A major current scientific challenge is scaling from the functional properties of organisms to processes at the ecosystem and global levels (Enquist et al. 2003; Torsvik and Ovreas 2002; Zak et al. 2006). Microbial respiration is a process that has particular importance in the ecosystem and global scales, representing about half of total CO2 flux from soils (Hanson et al. 2000). Furthermore, effects of human-induced climate change on soil microbial communities and their metabolic activities could create potentially devastating feedbacks to the Earth's biosphere (Meir et al. 2006). Biomass made up of fast-growing species respires faster than an equal amount of biomass made up of slow-growing species. Microbes with low growth yields (biomass produced per unit substrate consumed) convert a larger fraction of substrate into CO2 during growth, and so respire faster than efficiently growing organisms. It has been observed that there is an inevitable thermodynamic trade-off between growth rate and yield among heterotrophic organisms (Pfeiffer et al. 2001). Past authors have proposed that two opposing ecological strategies exist at either end of this spectrum: a fast-growing, low yield competitive strategy and a slow growing high yield cooperative strategy (Kreft and Bonhoeffer 2005; Pfeiffer et al. 2001). For microbes, the cooperative, slow, efficient growth strategy is more successful in spatially structured environments such as biofilms (Kreft 2004; Kreft and Bonhoeffer 2005; MacLean and Gudelj 2006; Pfeiffer et al. 2001). With over a billion individual cells and estimates of 104–105 distinct genomes per gram of soil (Gans et al., 2005; Tringe et al., 2005; Fierer et al., 2007b, David et al 2008), bacteria in soil are the reservoirs for much of Earth's genetic biodiversity. This vast phylogenetic and functional diversity can be attributed in part to the dynamic physical and chemical heterogeneity of soil, which results in spatial and temporal separation of microorganisms (Papke and Ward, 2004 Katherineel al 2011). Given the high diversity of carbon (C) – rich compounds in soils, the ability of each taxon to compete for only a subset of resources could also contribute to the high diversity of bacteria in soils through resource partitioning (Zhou et al., 2002). Indeed, Waldrop and Firestone (2004) have demonstrated distinct substrate preferences by broad microbial groups in grassland soils and C resource partitioning has been demonstrated to be a key contributor to patterns of bacterial co-existence in model communities on plant surfaces (Wilson and Lindow, 1994).

2. Theoretical Background

Void in the soil is the micropores that deposit in soil deposition at different structure and strata, the percentage of void in soil determine the tortuosity, and the this determined the level of flow path of fluid void in varies according to the type of soil under the influence of geological historical deposition, high degree of variation of void are deposition in soil base on this condition void ration in soil develop various percentage, the void ratio\ degree in organic and lateritic soli can never be the same with that of silty and coarse and fine sand formations. The flows under the influence of tortuosity will definitely vary, the condition implies that the hydraulic conductivity abound to be express variation, subject to this relation, the dispersion of fluid flow will also express similar variations under the influence of discrepancy of void and tortuosity, the flow of solute in like manner develop the rate of migration under the influence of the structural stratification of the void, the deposition of phosphorus in most cases are solute to fluid in soil, the deposition can at any formation of the soil, the formation of the soil determine the rate concentration. Moreso the structural deposition of the formation influence the variation of migration under the law of plug flow, such condition in the deposition of salmonella become subject of concern in the transport behaviour from one stratum to another, the rate of influence are determined by the degree of deposition of the microelement in the formation, in like manner to fluid flow under the influence of void ration the through the rate of tortuosity in the strata dispersion influence are recorded in the formation base on the rate void ration under the deposition of the formation through the geologic history in the study area. Phosphorous deposition in soil are determine on several conditions, but high concentration of salmonella are influence by the rate of phosphorus deposition in the formation, the microelements is a substrate to microbes, therefore there will be increase in salmonella population in the formation, to monitor the rate of phosphorus deposition under the influences of void ration and dispersion, mathematical model were found necessary to monitor the deposition of the microelement in soil and determine the rate of void ratio and dispersion influence on its deposition in the study location. The developed model was generated from a formulated equation to monitor the rate of void ratio and dispersion on phosphorous deposition influence on salmonella in the study area, the express governing equation is stated bellow.

3. Governing Equation

$$V\frac{\partial^2 c}{\partial t^2} = \frac{\partial cs}{\partial x} q_z C_s + Ds\frac{\partial cs}{\partial x} - M_b \frac{\mu_o}{\gamma_o}\frac{\partial c}{\partial x} + \frac{\partial cs}{\partial t}\frac{Cs}{K_{Ao} + Cs} + \frac{\partial cs}{\partial x}\frac{Cs}{K_A + C_A} \quad \ldots\ldots \quad (1)$$

$$\frac{\partial^2 c}{\partial t^2} = S^2 C_{(t)} - SC_{(t)} - C_{(o)} \quad \ldots\ldots\ldots\ldots \quad (2)$$

$$\frac{\partial cs}{\partial x} = SC_{(x)} - C_{(x)} \quad \ldots\ldots\ldots\ldots \quad (3)$$

$$\frac{\partial cs}{\partial x} = SC_{(x)} - C_{(x)} \quad \ldots\ldots\ldots\ldots \quad (4)$$

$$\frac{\partial cs}{\partial x} = SC_{(x)} - C_{(x)} \quad \ldots\ldots\ldots\ldots \quad (5)$$

$$\frac{\partial cs}{\partial t} = SC_{(t)} - C_{(t)} \quad \ldots\ldots\ldots\ldots \quad (6)$$

$$\frac{\partial cs}{\partial x} = SC_{(x)} - C_{(x)} \quad \ldots\ldots\ldots\ldots \quad (7)$$

Equation (1) to (7) were transform into Laplace, the expression is to transform the parameters to the form were it can be applied to express there various subject relation at there various function in the system, the transformation application are necessary because it streamline the parameters function at there various areas mathematically in the system, the expression also correlate the activities of the parameters that is establish to monitor the behaviour of the microelement under the influence of void ratio and dispersion of salmonella in the system.

$$V\left[S^2 C_{(t)} - SC_{(t)} - SC_{(0)}\right] + qzCs\left[SC_{(z)} - C_{(0)}\right] Ds\left[SC_{(z)} - C_{(0)}\right] -$$

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$$M_b \frac{\mu_o}{\gamma_o}\left[SC_{(z)} + C_{(0)}\right] + \frac{Co}{Ko+Cs}\left[SC_{(t)} + C_{(0)}\right] + \frac{CA}{KA+CA}\left[SC_{(z)} + C_{(0)}\right] \quad \ldots\ldots\ldots\ldots \quad (8)$$

$$V\left[S^2C_{(t)} - C_{(t)} - C_{(0)} + qzCs\right]\left[SC_{(z)}{}^2 - 2SC_{(z)}\left(C_{(0)}\right)^2\right]\left[M_b \frac{\mu_o}{\gamma_o} + 2SC_{(z)}C_{(0)} - \left(C_{(0)}\right)^2\right] \quad (9)$$

Equating (9) into time t, we have

$$V\left[S^2C_{(t)} - SC_{(t)} - SC_{(0)}\right] + qzCs\left[SC_{(z)}{}^2 - 2SC_{(z)}\left(C_{(0)}\right)^2\right] - \quad \ldots\ldots\ldots\ldots \quad (10)$$

$$M_b \frac{\mu_o}{\gamma_o}\left(SC_{(z)}\right)^2 - 2SC_{(z)}C_{(0)} + \left(C_{(0)}\right)^2 + \frac{Cs}{Ko+Cs}\left(SC_{(z)}\right)^2 - 2SC_{(z)}C_{(0)} + \left(C_{(0)}\right)^2 \quad \ldots \quad (11)$$

Equation (8) to (11) correlate various parameters by excising there various function in other to descretize it into various area the parameters activities are more severe, the function of the parameters are express in different conditions, such situation implies that the parameters activities may not be simultaneously in the system, these is base on the variation of the formation that influences all the parameters in the system, such situation implies that the parameters are influenced by the variation of the soil structural depositions, the expression from equation (8) to(11) were able to differentiate various parameters in accordance with the considered variations of the soil strata in the system, the expression were able to differentiates these parameters in line with there various function base on different condition considered in the study.

Rearranging (11) yield

$$a^2 - 2ap + p(a-b)^2$$

$$\left[1 + \frac{Cs}{Kso+Cs}\right]\left[SC_{(t)}\right]^2 - \left[1 + \frac{Cs}{Kso+Cs}\right]2SP_{(z)}C_{(0)} + \left[1 + \frac{Cs}{Kso+Cs}\right]\left[C_{(0)}\right]^2 \quad \ldots\ldots\ldots\ldots \quad (12)$$

$$\left[\left(SC_{(z)}\right)^2 - 2SC_{(x)}C_{(0)} + \left(C_{(0)}\right)^2\right]1 + \frac{Cs}{Kso+Cs} \quad \ldots\ldots\ldots\ldots \quad (13)$$

$$\left[\left(SC_{(t)}\right)^2 - 2SC_{(t)}C_{(0)} + \left(C_{(0)}\right)^2\right]1 + \frac{\frac{Cs}{KAo+CA}}{\frac{Cs}{Ks+Cs}} \quad \ldots\ldots\ldots\ldots \quad (14)$$

$$\left[SC_{(t)}\, C_{(0)}\right]^2 - \frac{\dfrac{CA}{KAo+CA}}{\dfrac{Cs}{Ks+Cs}} \qquad \ldots\ldots\ldots\ldots \quad (15)$$

Equation (12) to (15) express further on the activities of the parameters at different condition, but at this stage the concentration of the microbes salmonella were paramount in the system, because the behaviour of salmonella are determined by the rate of void ratio, dispersion deposition of phosphorous in the formation, therefore the concentration were thoroughly expressed in the system from equation (12) to (15) to monitor the behaviour with respect to time and distance travelled.

$$SC_{(x)} - C_{(0)} = \sqrt{\frac{\dfrac{CA}{KAo+CA}}{\dfrac{Cs}{Ks+Cs}}} = \pm 1 \sqrt{\frac{\dfrac{CA}{KAo+CA}}{\dfrac{Cs}{Ks+CA}}} \qquad \ldots\ldots\ldots\ldots \quad (16)$$

$$SC_{(x)} = C_{(0)} \pm 1 \sqrt{\frac{\dfrac{CA}{KAo+CA}}{\dfrac{Cs}{Ks+Cs}}} \qquad \ldots\ldots\ldots\ldots \quad (17)$$

$$SC_{(x)} = C_{(0)} + 1 \sqrt{\frac{\dfrac{CA}{KAo+CA}}{\dfrac{\dfrac{Cs}{Ks+Cs}}{S}}} \qquad \ldots\ldots\ldots\ldots \quad (18)$$

F(x) when $x > 0 \quad C_{(o)} = P_0$

$$SC_{(x)} = \frac{C_0}{S} + \sqrt{\frac{\dfrac{CA}{KAo+CA}}{\dfrac{\dfrac{Cs}{KA+CA}}{S}}} \qquad \ldots\ldots\ldots\ldots \quad (19)$$

Hence, in any direction of x, we have

$$C_{(x)} = \ell^{\frac{C_0}{S}} \left[ACos \sqrt{\frac{\frac{CA}{KAo+CA}}{\frac{\frac{Cs}{Ks+Cs}}{S}}} + BSin \sqrt{\frac{\frac{CA}{KAo+CA}}{\frac{\frac{Cs}{Ks+CA}}{S}}} \right] x \qquad \ldots\ldots\ldots\ldots \quad (20)$$

$$\Rightarrow \; C_{(x)} = \ell^{C_0 t} \left[ACos \sqrt{\frac{\frac{CA}{KAo+CA}}{\frac{\frac{Cs}{Ks+CA}}{S}}} \; t + BSin \sqrt{\frac{\frac{CA}{KAo+CA}}{\frac{\frac{Cs}{Ks+CA}}{S}}} \right] x \qquad \ldots\ldots\ldots\ldots \quad (21)$$

To monitor the system under exponential phase quadratic expressions were fine suitable , the parameters were found to influence the concentration of salmonella , therefore mathematical expression of quadratic equations were applied to express the parameter in exponential phase, the microelement is substrate to salmonella, this condition is were there is increase in microbial population, so at this condition there no tendency of degradation of the microbes, other influential parameters in the system are in continuous process thus influencing the deposition of phosphorus and salmonella in the study location, therefore the approach of applying quadratic expression were suitable in the derived mathematical expression.

Again, we consider (10), so that we have

$$V\left[S^2 C_{(t)} - SC_{(t)} - SC_{(0)}\right] + qzCs\left[SC_{(z)}{}^2 - 2SC_{(z)}\left(C_{(0)}\right)^2\right]$$

$$V\left[S^2 C_{(t)} - SC_{(t)} - C_{(0)}\right] = -qzCs\left[SC_{(z)}{}^2 - 2SC_{(z)}\left(C_{(0)}\right)^2\right] \qquad \ldots\ldots\ldots\ldots \quad (22)$$

$$\frac{S^2 C_{(t)} - SC_{(t)} - C_{(0)}}{\left(SC_{(t)} - C_{(0)}\right)^2} = \frac{qzCs}{V} \qquad \ldots\ldots\ldots\ldots \quad (23)$$

$$SC_{(t)} - C_{(0)} \neq 0 \qquad \ldots\ldots\ldots\ldots \quad (24)$$

Considering the LHS of the numerator of (23) gives

$$C_{(t)} = \frac{S \pm \sqrt{S^2 + 4S^2{}_{(o)}}}{2S^2} \qquad \text{...........} \quad (25)$$

$$C_{(t)} = \frac{1}{2S}\frac{\pm\sqrt{1+4_{(o)}}}{2S} \qquad \text{...........} \quad (26)$$

When $t > 0 \quad C_{(o)} = C_0$

So that $C_{(t)} = \frac{1}{2S}\frac{\pm\sqrt{1+C_o}}{2S}$

$$C_{(t)} = A\ell^{\frac{1}{2}\left(1+\sqrt{1+C_o}\right)t} + B\ell^{\frac{1}{2}\left(1-\sqrt{1+C_o}\right)t} \qquad \text{...........} \quad (27)$$

Since the denominator of the LHS of (23) has equal Roots

$$P_{(t)} = -\frac{qzCs}{V}(C+Dt)\ell^{\,(t\;-P_o)t} \qquad \text{...........} \quad (28)$$

Combining equation (27) and (28), we have

$$C_{(t)} = -\frac{qzCs}{V}(C+Dt)\ell^{\,(1\,+\,C_o)t} + A\ell^{\frac{1}{2}\left(1+\sqrt{1+C_o}\right)t} + B\ell^{\frac{1}{2}\left(1-\sqrt{1+C_o}\right)t} \qquad \text{...........} \quad (29)$$

The expression from equation (22) to (29) were to express the parameters at various area where they are more influential on the deposition of phosphorous and salmonella in the formation, these equations were able to descretize the parameters from equation (10), the expression in equation ten were recalled in other to express the activities of parameters at various level, there various role of influence on the system were expressed at different condition were they influence the deposition of phosphorous and dispersion in the system\, subject to this relation, the behaviour of the microelements has displayed the consequences through the various activities and function of the parameters. The behaviour of phosphorous are is influenced by the depended variables in the system; the expression from this dimension integrated the parameters by denoting through mathematical symbols, the express equation in (29) shows that the deposition of phosphorous and concentration of salmonella are with respect to time, the influence from void ration developed the spread of the phosphorous and salmonella in the study area, the expression with respect to time are stated below.

If $t = \frac{x}{V}$

$$P_{(x,v)} = A\ell^{\frac{1}{2}\left(1+\sqrt{1+C_o}\right)\frac{x}{v}} + B\ell^{\frac{1}{2}\left(1-\sqrt{1+C_o}\right)\frac{x}{v}} - \frac{qzCs}{V}(C+Dt)\,\ell^{\,(1\,+\,C_o)\frac{x}{v}} \qquad \text{...........} \quad (30)$$

The final model equation shows the concentration of salmonella and the deposition of phosphorous considered with respect to distance , the final expressed model velocity of transport were considered, the velocity of transport were express in the system were other influential parameters are denoted by other mathematical symbol the major parameters that determine the deposition of phosphorous an salmonella is void ratio of the soil, it is the tortuosity are established which determine the flow path in the soil formations, the parameter were expressed as the paramount variables in the final model expression. The formation from this dimensions are thoroughly expressed in the final model equation, the model show the rate of exponential phase, because an increase in degree of void ratio determine the rate of tortuosity in soil formation the condition may definitely increase the rate velocity of flow in fluid on soil formations, therefore the influence of void ratio are subject to increase dispersion influence of salmonella deposition in the study location.

4. Conclusion

The rate of void ratio in soil has been thoroughly express in the derived model equation, the rate of void ratio depend on the soil structural deposition through the geologic history in the study location. The deposition of the microelements in the system are determined on the degree of void ratio the soil the degree of void ratio in the study location are confirmed through standard experiment of void ratio to be in a very high degree , such condition are reflected on the rate tortuosity that experience high degree also in the system, the expression in this condition generate high velocity of flow, the condition implies that the transport of salmonella and phosphorous will be very fast under the influence of high degree of void ratio deposition in the study area. The area under study is in deltaic environment, from the investigation carried out the rate of void are very high at every part of the study area, this condition implies that fast migration of microelements abound to be experienced in the formations. The expressed final model will able to streamline the behaviour of the substrate deposition and the migration of salmonella in the formation, the expression from the final model shows that there is the tendency of continue increase of salmonella in the study area, this impression are from the fact that there is high deposition of phosphorous in the formation such condition is of serious concern in soil and water environment, the developed model will definitely monitor the degree of void ratio and the deposition of phosphorous on the influence of salmonella in the study location.

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